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# THEORETICAL INVESTIGATION OF EM WAVE GENERATION AND RADIATION IN THE ULF, ELF, AND VLF BANDS BY THE ELECTRODYNAMIC ORBITING TETHER

GRANT NAG8-638

Semiannual Report #1

For the period 1 May 1987 through 31 October 1987

Principal Investigator

Dr. Robert D. Estes

April 1988

Prepared for
National Aeronautics and Space Administration
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# 1.0 INTRODUCTION

The goal of this project is to extend our previous analysis of electromagnetic wave generation by an electrodynamic tethered satellite system to a more realistic model that includes the effects on wave propagation and reflection of the boundaries between ionosphere, atmosphere, and Earth. The previous work considered an infinite, uniform medium, which was a reasonable place to begin, especially from the standpoint of understanding the mechanism by which electromagnetic waves are generated and the electrical circuit of the electrodynamic tethered satellite is completed in the ionosphere. Reality is much more complicated, however, and to estimate the strength of and otherwise characterize the electromagnetic waves reaching the Earth's surface requires a more complex model. Unfortunately, there is no "off the shelf" analysis available. Although a number of investigators have used computer codes based on a point or finite dipole source to represent the tether, we have decided that all such analyses are missing fundamental points about the way an electrodynamic tethered satellite system works. Certainly in its steady-current mode of operation it does not radiate as an antenna. It excites—as a generator—transmission line waves in the ionosphere. Thus blind application of available theories is not useful. The problem of the tether as an antenna will be another order of magnitude more difficult than a more realistic model of a tether with steady-state or slowly varying current. For a frequency range that has yet to be determined, there will be a combination of "ordinary" transmitting antenna radiation and tether transmission line wave excitation. One of the problems that we intend to address in the course of this study is what a variation in the tether current does to the Alfvén wings and the electromagnetic field variations experienced at a distance from the tethered system. Extending the boundary problem analysis to time-varying currents is another goal of our analysis.

We begin our report by reviewing our previous analysis and how it has been received by other investigators in the field. This seems important to do since the earlier analysis forms a starting point for the extended analysis. When we finished the Smithsonian Astrophysical Observatory (SAO) work under NASA Grant NAG8-551, we felt we had made a significant contribution to understanding electromagnetic wave generation by an electrodynamic tethered satellite system. We had described the Alfven wings with more detail than previous analyses. More importantly from the standpoint of tether applications, we had shown that modeling the tethered system as an orbiting wire was not reasonable and had produced very misleading results, which were excessively pessimistic about the possibility of drawing substantial currents in the tether. This point now appears to have been generally accepted. There is, however, still disagreement with a group of Italian investigators; Dobrowolny and Veltri carried out an analysis of Alfvén wave generation by a tethered system and obtained a functional dependence of the wave impedance on the satellite velocity and system dimensions quite different from ours, and they still believe it to be correct. Although we were confident in our results and had pointed out what we considered to be flaws in the physical reasoning used by Dobrowolny and Veltri to justify their results, our respect for their opinions convinced us that making a more careful examination of this disagreement was the wisest course to follow, given the importance of our earlier analysis to what we planned to do. The next section is mainly devoted to this question. We emerged from this exercise more firmly convinced of the correctness of our previous results than ever, having rederived them in two different ways. Not only that: we found an apparent inconsistency in their deviation of the wave impedance.

One of our major activities in this first reporting period has been searching the scientific literature for publications that might be relevant to our problems. In Section 3 we list some of the more useful papers we have found and read (some more completely than others, of course). It appears that there is no "off the shelf" solution to be found in the literature either, but a number of theoretical studies have suggested approaches that might be fruitful.

Section 4 describes software we have developed at SAO to follow the path of waves along field lines through the ionosphere to the atmosphere starting from an arbitrary position in the atmosphere. We present some preliminary results from applying the code to the location of wave reception "hot spots" on the Earth's surface for satellites operating at 300 km and 600 km altitudes.

Section 5 presents a generalization of the Alfven wing analysis to allow for arbitrary angles between the velocity vector, geomagnetic field, and the vertical. This will be utilized in our modeling of the problem with boundaries included.

Section 6 briefly sketches the rest of the work we will be undertaking.

## 2.0 UPDATE AND STATE OF THE CONTROVERSIES ON TETHER WAVE GENERATION

The conclusions presented in the Smithsonian Astrophysical Observatory (SAO) study of electromagnetic wave generation that preceded the one in progress can be re-examined now that they have been publicized through the circulation of the final report and preprint and through oral presentations made by R. Estes, the Principal Investigator for the two studies, in a number of forums. A review of the results and their reception by the community of investigators in the field seems a good place to begin the summary of the first six months of our new investigation, since we are building on those earlier results. This is especially true since one group, whose own results were at odds with those of the SAO study, has not been convinced that the SAO results are correct.

The theory developed in the prior study is reported in detail in the final report (NAG8-551). A slightly modified version of the portion of the report that deals with the Alfvén wings and the general subject of electromagnetic radiation from an electrodynamic tethered satellite system will appear in The Journal of Geophysical Research. The study built upon previous ones, but sought to obtain more believable results by using a more realistic model for the electric current distribution of the electrodynamic tether. Previous investigators had modeled the tether current system as a long cylinder. They had either considered an orbiting canister with a dimension along the direction of flight measured in tens of meters or, at the other extreme, an orbiting wire. Neither approach took into account the peculiar dumbbell shape of a tethered satellite system, which consists of a long, narrow wire terminated by satellites (or plasma clouds) with dimensions along the direction of motion that are much greater than the tether diameter. The SAO study showed that the orbiting canister approach, although it had not been thoroughly justified before, should provide a reasonable approximation for the purposes of calculating the wave impedance and ionospheric currents associated with steady-state (constant current) operation of an electrodynamic tethered satellite system, provided that the current distribution assumes a constant value along the length of the tether, i.e., an insulated tether is assumed. This is true because it is primarily the changing fields associated with the injection of charge into the ionosphere at the ends of the moving system that drive the electromagnetic waves. Thus, any two moving systems that inject charge into the ionosphere at the same rate over the same area are equivalent from the standpoint of wave generation. By the same token, the orbiting wire model was found to be a poor model for an electrodynamic tether because it undervalued the dimensions of the terminating, charge-exchanging parts of the system by orders of magnitude.

Barnett and Olbert [1986] of MIT had used calculations based on an orbiting wire model to advance the idea that the wave impedance associated with the operation of a constant current electrodynamic tethered satellite system would be on the order of 10,000 to 100,000 Ohms, as opposed to previous estimates on the order of an Ohm. This was not an academic question, since such high impedance values would have precluded the use of tethered systems for any applications that required substantial currents. This conflict has now been resolved, and Barnett and Olbert have acknowledged that their use of the orbiting wire model led to a great overestimation of the wave impedance associated with the frequency band lying between the lower hybrid frequency and the electron cyclotron frequency. In a private communication they expressed complete agreement with the SAO study on this point, and in a new study, also slated for publication in The Journal of Geophysical Research, they credit R. Estes with pointing out that it is only the system/ionosphere charge-exchange region that matters for wave excitation within the steady-state, cold plasma models considered up until now.

Agreement has not yet been reached with M. Dobrowolny and his colleagues on a number of points, which involve the functional dependence of the Alfvén wave impedance on the Alfvén and satellite velocities as well as the system dimensions. The SAO study, in agreement with all other studies but one, found that this impedance varies linearly with the Alfvén speed, while being independent of the satellite velocity within the approximations used. Dobrowolny and Veltri [1986], however, had found an inverse dependence on the Alfvén speed and a quadratic dependence on the satellite speed. They have based their defense of their results largely on the thesis that the quadratic dependence on the satellite speed is much more reasonable physically, since the effect disappears (as it should) if the system is not moving. We will discuss this in more detail below. Their original paper also contained an argument to explain the divergence of their results from those in the 1965 work of Drell, et al. We have already criticized that physical argument. They did not address the question of why their results differed from all of the other prior studies as well, studies which obtained quite different results from those of Drell, et al. Nor did they discuss why the Drell formula's linear variation with the tether length and inverse variation with the system's dimension along the direction of motion should apply to the case they had considered at all, since Rasmussen et al. [1985] had already pointed out that the Drell results did not apply to a tethered system. In fact, Chu and Gross in 1966 ascertained that the Drell results were not applicable to any system having a vertical dimension that was not much smaller than its dimension along the direction of motion, i.e. just the opposite situation from what applies in the case of a tethered satellite.

Basically, Dobrowolny and his colleagues maintain that the bulk of the plasma current is a "dc" current having nothing to do with wave phenomena. We argue that this amounts to saying that a current pulse traveling down a transmission line is a dc current. This approach appears fundamentally wrong when dealing with plasma currents, which are essentially wave phenomena.

Although Dobrowolny and his co-workers have not emphasized this part of their results, the linear dependence of the Alfvén wave impedance on the tether length that they found is as important a difference from the results of other investigators as the inverse dependence on the Alfvén speed is.

Since Dobrowolny and his group are long-time investigators in the field of tether electrodynamics and have remained firmly convinced that their results are correct, and since these results would require considerable re-evaluation of what we have heretofore thought we understood about constant current electrodynamic tethered satellite systems, it seemed prudent to consider their analysis and their criticism of the SAO analysis in some detail. Thus we have devoted considerable time to a more detailed examination of their analysis than we did originally. We emphasize that as regards their criticism of our results we will be summarizing verbal remarks Dobrowolny and Iess have made rather than referring to anything they have written. We believe we have understood their criticisms and will attempt to make a fair presentation of them, while stating from the outset that their arguments have not convinced us. We have in fact rederived our results using their formulation of the problem.

First let us consider their original analysis (as presented in the Nuovo Cimento article by Dobrowolny and Veltri) and compare it with the SAO analysis. Dobrowolny and Veltri use a long orbiting cylinder with square cross-section to model the tethered system. Since *D*, the tether's transverse dimension, is stated to be on the order of 1 mm, it is clear that this is just another variation of the orbiting wire model, the inadequacy of which we have previously demonstrated. However, the authors do not deal at all with the issue of radiation in other frequency bands raised by Barrett and Olbert. Following a different approach from that of the SAO study, they begin by writing down the most general, allinclusive formula for the power radiated by a current distribution in a plasma (equations (9)-(12) in the original):

$$P = \lim_{T \to \infty} \frac{1}{2\pi^2 T} \int d^3k \int \frac{d\omega}{\omega} \lambda_{ss} (\vec{k}, \omega) \left| J_j^{(e)*}(\vec{k}, \omega) e_j(\vec{k}, \omega) \right|^2 \delta(\Lambda) \qquad (1)$$

where

$$\lambda_{ss} = Tr \lambda_{ij} \qquad (2)$$

and $\lambda_{ij}$ is the co-factor of the plasma tensor $\Lambda_{ij}$ defined by

$$\Lambda_{ij} = n^2 \left( \frac{k_i k_j}{k^2} + \delta_{ij} \right) + \epsilon_{ij}(\vec{k}, \omega) \qquad (3)$$

using standard notation. Finally $\Lambda = det\ \Lambda_{ij}$, and $e_j$ is the polarization vector defined by

$$e_i = \frac{\lambda_{ij} a_j}{\left( \lambda_{ss} \overset{*}{a_i} \lambda_{ij} a_j \right)^{1/2}} \qquad (4)$$

in terms of an "arbitrary complex vector $a$." $\vec{J}^{(e)}$ is the external current, which we have denoted by $\vec{j}$. Neither at this point nor at any later point do the authors ever specify the form of $\epsilon_{ij}$. They maintain a high level of abstraction. The meaning of the polarization vector seems particularly obscure to us in this context.

For formula (1) the authors refer to Plasma Astrophysics: Nonthermal Processes in Diffuse Magnetized Plasmas, Volume 1 by D.B. Melrose. In fact the expression does not appear precisely this way in the reference. What Melrose writes is (equation (3.8) p. 65)

$$P = -\lim_{T \to \infty} \frac{1}{T} \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \left[ \vec{J}^{(e)*}(\vec{k},\omega) \cdot \vec{E}(\vec{k},\omega) \right] \qquad (5)$$

where we have slightly altered the notation to be consistent with Dobrowolny et al. The integrand in Melrose is the dot product of the Fourier transform of the wave electric field vector and the complex conjugate of the Fourier transform of the external current density, which is the tether current in our case.

Before continuing with our discussion of the analysis of Dobrowolny, et al. let us apply this equation, i.e. the equation as it actually appears in Melrose, to the SAO results.

We have already seen that the contribution of the $j_x$ component to the radiated power is small, so we consider the contribution of $j_y{}^*E_y$. Referring to the previous SAO results, we obtain

$$j_y{}^* E_y = \frac{4\pi i\omega}{k_\perp^2 c^2} \frac{\vec{k} \cdot \vec{j}}{k_z{}^2 - \omega^2 \epsilon_\perp / c^2} k_y j_y{}^*$$

$$= \frac{8I^2 i\omega}{\pi c^2} \frac{\left[\delta(\omega - k_x v_x)\right]^2 \sin^2\!\left(k_y L/2\right) \sin\!\left(k_x L_x/2\right)}{k_\perp{}^2\left(k_z{}^2 - \omega^2 \epsilon_\perp / c^2\right) k_x L_x} \tag{6}$$

Using equation (2.12) (p. 27) of Melrose we can make the substitution

$$\left[\delta(\omega - k_x v_x)\right]^2 = \lim_{T \to \infty} \frac{T}{2\pi} \delta(\omega - k_x v_x) \tag{7}$$

Then we obtain the expression for the power going into waves

$$P = -\frac{4I^2 i}{L_x \pi^2 c^2} \int d\omega\, d^3k \; \frac{\omega \;\delta(\omega - k_x v_x) \sin^2(k_y L/2) \sin(k_x L_x/2)}{k_\perp{}^2 \cdot (k_z{}^2 - \omega^2 \epsilon_\perp / c^2) \;\; k_x} \tag{8}$$

where factors of $2\pi$ have disappeared because we use a different convention from Melrose's in defining the Fourier transform. Performing the integration over $\omega$ and $k_z$ exactly as before, we get

$$P = \frac{4I^2 v_A}{\pi L_x c^2} \int_{-K_O}^{K_O} dk_x \int_{-\infty}^{\infty} dk_y \; \frac{\sin^2(k_y L/2) \sin(k_x L_x/2)}{k_\perp{}^2 k_x} \; \sqrt{1 - \left(\frac{k_x v_x}{\Omega_{ci}}\right)^2} \tag{9}$$

Utilizing the formula

$$\int \frac{\sin^2 ax}{x^2 + \beta^2}\, dx = \frac{\pi}{4\beta}\left(1 - e^{-2a\beta}\right)$$

$$\left(a > 0,\ Re\beta > 0\right) \tag{10}$$

found in Gradshteyn and Ryzhik's tables (p. 447), we finally obtain

$$P = \frac{I^2 v_A}{L_x c^2} \int_0^{K_0} dk_x \left(1 - e^{-k_x L}\right) \frac{\sin(k_x L_x/2)}{k_x^2} \sqrt{1 - \left(\frac{k_x v_x}{\Omega_{ci}}\right)^2} \tag{11}$$

Dividing by $I^2$ to obtain the impedance $Z_A$, we find we have arrived at an expression identical to the one derived previously in the SAO study. This exercise has demonstrated that by starting from the same point as Dobrowolny et al. we still arrive at the same result as previously. Since expression (5) is completely general, this had to be, assuming we had made no errors.

The purpose of this exercise has been twofold: first to point out that if there are errors in the SAO analysis they must have occurred in the derivation of the expression for $E_k$. Since this derivation is easily followed, it should be possible to discover any errors made there. The point that follows from this is that we should be doubly careful in evaluating the analysis of Dobrowolny et al. which supposedly uses the same starting point.

The SAO analysis made use of the $E_z = 0$ approximation to good purpose. When this is not done, a great deal of unnecessary apparent complexity remains, with a concurrent increase in the probability that errors in calculations or physical reasoning will occur.

Let us turn our attention to equation (1), the starting point for the analysis of Dobrowolny and Veltri. We have already mentioned that it is itself a derived expression not found explicitly in the Melrose reference. Although the authors have not demonstrated the intermediate steps, we can attempt to reconstruct their chain of reasoning.

Melrose gives the general equation ((3.16), p. 67)

$$E_i(\vec{k},\omega) = \frac{-4\pi i}{\omega} \frac{\lambda_{ij}(\vec{k},\omega)}{\Lambda(\vec{k},\omega)} J_j^{(e)}(\vec{k},\omega) \tag{12}$$

This follows from the definitions and the basic wave equation.

Inserting this into equation (5) gives

$$P = \frac{-\lim}{T \to \infty} \frac{4\pi i}{T} \int \frac{d\omega}{2\pi\omega} \int \frac{d^3k}{(2\pi)^3} \frac{J_i^{(e)}\lambda_{ij}J_j^{(e)*}}{\Lambda} \tag{13}$$

Now for a given mode $\sigma$, Melrose defines the polarization vector $e_i^{\sigma}$ in the way Dobrowolny and Veltri seem to be doing for all modes, inclusively, in expression (4). We have not examined this issue thoroughly, but it is not immediately obvious to us that this extension makes sense, i.e. that a single arbitrary complex vector $\vec{a}$ would yield physically meaningful results for all modes. Granting this point (They never say what $\vec{a}$ they use.), we then obtain

$$\lambda_{ij} = \lambda_{ss} \ e_i e_j^* \tag{14}$$

which leads to

$$P = \lim_{T \to \infty} - \frac{i}{4\pi^3 T} \int \frac{d\omega}{\omega} \int d^3k \ \frac{\lambda_{ss}}{\Lambda} \ \left| \vec{J}^{(e)*} \cdot \vec{e} \right|^2 \tag{15}$$

This becomes identical with equation (1) if we make the substitution

$$\frac{1}{\Lambda(\vec{k}, \omega)} = 2\pi i \ \delta\Big( \Lambda(\vec{k}, \omega) \Big) \tag{16}$$

At this point, the meaning of $\delta(\Lambda)$ is somewhat ambiguous. Although the authors present the next step in their results without any explanation beyond a characterization of the intermediate calculations as "lengthy", we gather from the factor $\left| \partial\Lambda / \partial\omega \right|^{-1}$ appearing in their result that they are taking the $\delta(\Lambda)$ to mean that $\Lambda$ is a function of $\omega$, which depends parametrically on $\vec{k}$ insofar as the integration over $\omega$ is concerned. Thus the integration gives a sum of terms from the poles of $\Lambda^{-1}$ (i.e. the zeros of $\Lambda$). These poles would correspond to the solutions of the dispersion relation $\Lambda(\omega, \boldsymbol{k}) = 0$. Therefore

$$\delta(\Lambda) = \sum_{\sigma} \delta\Big(\omega - \omega_{\sigma}(\boldsymbol{k})\Big) \Big/ \left| \frac{\partial \Lambda}{\partial \omega} \right|_{\omega=\omega_{\sigma}(\boldsymbol{k})}$$

where the sum is over the different wave modes.

Since they were only going to consider Alfvén waves, the authors could have saved themselves a lot of computational labor by continuing to follow the analysis presented in Melrose's book. On page 56 of that work we find that following results

$$\vec{e}^{A} = \frac{\vec{k}_{\perp}}{k_{\perp}} \tag{17}$$

$$\frac{\lambda_{ss}^{A}(\vec{k})}{\left[\,\omega\, \partial\Lambda \,/\, \partial\omega\, \Lambda\,\right]_{\omega=\omega_A}} = \frac{V_A{}^2}{2c^2} \tag{18}$$

where $\vec{e}^{A}$ is the Alfvén wave polarization vector. (Refer to page 36 for the relevant coordinate axes.)

Equation (15) thus becomes

$$P_A = \lim_{T \to \infty} \frac{1}{2\pi^2 T} \int d^3k \int_{-\omega_o}^{\omega_o} d\omega \, \frac{\lambda_{ss}}{\left(\omega \, \partial \Lambda / \partial \omega\right)_{\omega = \omega_A}} \, \delta\left(\omega - \omega_A(\vec{k})\right) \frac{J_y^{(e)2} k_y^{\,2}}{k_\perp^2} \tag{19}$$

$$= \lim_{T \to \infty} \frac{V_A^2}{4\pi^2 T c^2} \int d^3k \int_{-\omega_o}^{\omega_o} d\omega \; \delta\left(\omega - \omega_A(\vec{k})\right) \frac{J_y^{(e)2} \; k_y^{\,2}}{k_\perp^{\,2}} \tag{20}$$

where $\omega_o$ is a cut-off frequency such that $\omega_o \ll \Omega_{ci}$.

Continuing in much that same way as in the calculation immediately preceding this one and using the Dobrowolny/Veltri expression for $J^{(e)}$, we obtain

$$P_A = \frac{2 I_o^{\,2} V_A}{\pi c^2} \int_{-K_o}^{K_o} dk_z \int \frac{dk_y}{k_\perp^{\,2}} \frac{\sin^2(k_y L/2) \sin^2(k_z V_o D/2 V_A)}{(k_z V_o D/2 V_A)^2 (k_z D/2)^2} \sin^2\left(\frac{k_z D}{2}\right) \tag{21}$$

where $K_o = \omega_o / V_o$.

We can certainly assume $\left(\frac{\mathrm{k_x V_o D}}{\mathrm{2 V_A}}\right) \ll 1$. The integral over $k_y$ has been done in the preceding calculation. We can apply that result to obtain

$$P_A = \frac{I_o^{\,2} V_A}{c^2} \int_0^{K_0} dk\,x \left(1 - e^{-k_z L}\right) \frac{\sin^2(k_z D/2)}{k_z (k_z D/2)^2} \tag{22}$$

Since $K_o \ll \Omega_{ci} / V_o$ , we can take $K_o D \ll 1$ so long as $D \le 25 m$. Then

$\frac{\sin(k_z D/2)}{(k_z D/2)} \approx 1$ throughout the range of integration, which gives us

$$P_A = \frac{2I_o^2 V_A}{D\ C^2} \int_0^{K_0} dk_z \left(1 - e^{-k_z L}\right) \frac{\sin(k_z D/2)}{k_z^2} \tag{23}$$

Again we have arrived at the very same expression as before (equation (11)) in the limit where $k_z D \ll 1$ except for a factor of 2. This factor could be due to a mistake we've made, but the more likely source is from equation (16) which we inferred from the Dobrowolny/Veltri expression (1).

These authors took an approach different from the one followed above. They attempted to carry through the complicated calculations using the general expression for $\Lambda$, $\lambda_{ij}$, and $\vec{e}$, only applying the Alfvén approximation at a later stage in the process.

Having written down the general formula (1), the authors then proceed to give results based on calculations which they characterize as "lengthy," too lengthy, evidently, to be summarized even in an appendix. Thus there is no way to follow their calculations step by step in order to judge this part of their work. They obtain (equations (13)-(15) in the original):

$$P = \frac{32I_o^2}{\pi D^4} \sum_\sigma \int d^3k \int \frac{d\omega}{\omega} H\ (k,\theta,\varphi) G(k,\theta,\delta,\omega) \delta\Big(\omega - k_x V_o\Big) \bullet$$

$$\bullet \left| \frac{\partial \Lambda}{\partial \omega} \right|^{-1} \delta\big[\, \omega - \omega_\sigma(k) \big] \quad \frac{1}{k_x^2 k_y^2 k_z^2} \tag{24}$$

where

$$H(k,\theta,\varphi) = \sin^2\!\Big(k_x \frac{D}{2}\Big) \sin^2\!\Big(k_y \frac{L}{2}\Big) \sin^2\!\Big(k_z \frac{D}{2}\Big) \tag{25}$$

$$G(k,\theta,\varphi,\omega) = \left[\epsilon_1\epsilon_2 - n^2\Big(\epsilon_3 \cos^2\theta + \epsilon_1 \sin^2\theta\Big)\right] \cos^2\varphi +$$

$$+ \epsilon_2 \sin^2\varphi \frac{\big[\epsilon_3 + n^2 \sin^2\theta\big]^2}{\epsilon_1\epsilon_2 - n^2\Big(\epsilon_3 \cos^2\theta + \epsilon_1 \sin^2\theta\Big)} \tag{26}$$

In formula (5), "the notation $\sum_\sigma$ denotes a summation over all plasma modes which are possible in the cold-plasma approximation, $\omega = \omega_\sigma(k)$ being the dispersion relation for the $\sigma$ mode."

At this stage of their calculation, they insert the Alfvén wave dispersion relation into their equations. Equation (24) appears to say that for every value of $\omega$ there is a reasonable solution to the dispersion relation for the $\sigma$ mode. This does not make physical sense, since they clearly view the Alfvén wave as a mode.

The formal development appears seriously flawed to us.

There are two (typographical ?) errors in the condition obtained for the angle $\theta$ that $\vec{k}$ makes with the $z$ axis. The authors quote $\theta = \arctan\left(\frac{V_o}{V_A \cos\phi}\right)$ and say that $V_A/V_o \ll 1$ implies $\theta \approx \pi/2$. In fact, $\theta = \arctan\left(\frac{V_A}{V_o \cos\phi}\right)$, and the assumption (true for TSS) is $V_A/V_o \gg 1$. The expression they obtain for the Alfvén wave power is

$$P_A = \frac{32 I_o^2}{\pi D^4} \frac{V_o}{V_A} \int_o^\infty k^2\, dk \int_o^\pi \sin\theta\, d\theta \int_{-\pi/2}^{\pi/2} d\theta \quad \frac{1}{\omega_A(k)}$$
$$\left\{ H(k,\theta,d) \cdot G(k,\theta,d,\omega_A(k)) \cdot \left[ \delta(\theta-\theta_1) + \delta(\theta-\theta_2) \right] \right\} \tag{27}$$

The Alfvén dispersion relation is only valid for frequencies much less than the ion cyclotron frequency. Thus for the power calculation to make sense as a calculation of power into Alfvén waves, where the Alfvén dispersion relation is inserted into the integrand, the integral over $\omega$ in Equation (24) must be restricted to a range $|\omega| \ll \Omega_{ci}$. We note that there seems to be another typographical error in equation (27). Presumably a factor $\left[k_x^2 k_y^2 k_z^2 \partial\Lambda/\partial\omega\right]^{-1}$ should be included in the integrand. The more important point is that the integration over all $\omega$ has been carried out and it has not placed any limitation on the size of $k_z$ in the integrand. If the rest of the integrand were such that only small values of $k_z$ were important, this would not be significant; but in the orbiting wire approximation, which is later carried further to an orbiting mathematical line approximation, this cannot be valid.

At this point the authors decide to introduce the restriction on $k_z$. The correctness of this step is more difficult to ascertain because of the likelihood of a typographical error in the preceding step. We did not attempt to reconstruct the calculations in full, since it would have required too many guesses about what expressions the authors were using for the dielectric tensor, etc. In any case the next step involves "very lengthy" calculations, of which no details are reported.

It is only at this stage of their calculations that Dobrowolny and Veltri begin to make approximations about the system dimensions. Their approximations are rather extreme and certainly do not correspond to a real tethered satellite system. Despite their statement near the beginning of the paper to the effect that the earlier work of Belcastro, Dobrowolny and Veltri [1982] had missed the Alfvén wings entirely because it had assumed a tether of infinite length, they proceed to make the same approximation. Not only that—they also make the extreme approximation of an orbiting wire with negligible radius. This approximation is also at odds with their previous statement that the finite size of the system was of essential importance. The point is that, by making these approximations at this later stage in the analysis, the authors have made it difficult to judge their effects. The approximations do not seem to be consistent. At one point they take $\bar{D} \equiv$

$D\Omega_{ci}/V_o \ll 1$. Later they take $\xi_m \sim 1/\bar{D}$, which would imply $\xi_m \gg 1$, when the Alfvén condition is $\xi_m \ll 1$.

Using the $\bar{D} \ll 1$ limit (orbiting wire) they had arrived at the result ((35) in their paper)

$$P_A = \frac{I_o^2 V_o^2}{c^2 V_A} \bar{L} \xi_m \left( \frac{1}{3} \xi_m^2 + \frac{V_A^2}{c^2} \right) \qquad (28)$$

They then take $\xi_m \sim \frac{1}{\bar{D}}$.

Now $\xi_m$ is the maximum value of $k_z V_o/\Omega_{ci}$ allowed for the Alfvén dispersion relation to apply. It is thus necessarily much less than 1. Taking $\xi_m \sim \frac{1}{\bar{D}}$ violates that condition in the extreme. They then take $\xi_m \sim 1$ to obtain

$$R_A = \frac{2}{3} \left( \frac{V_A}{c^2} \right) \left( \frac{V_o}{V_A} \right)^2 \frac{L}{D} \qquad (29)$$

as the Alfvén wave impedance.

We note that they calculated the Alfvén wave impedance at around $1\Omega$ <u>before</u> this slight of hand involving the use of $\xi_m \sim \frac{1}{\bar{D}}$.

Let us take their final expression (29) for this wave impedance and apply it to the 100 km long tethered system with a 1 mm diameter tether.

Since $L/D = 10^8$, $\frac{1}{3}\left(V_o/V_A\right)^2 \simeq 2\times 10^{-4}$, and $\frac{2V_A}{c^2}$ corresponds to around $0.06\Omega$, we find $R_A \simeq 1200\Omega$!

Thus the authors seem to have made a serious mistake in obtaining their expression for the Alfvén wave impedance. Perhaps they meant to apply $\xi_m \sim 1/\bar{D}$ only to the case for which $\bar{D} \gg 1$ (though they should have said so). Then they would have obtained

$$R_A = \frac{2}{3}\frac{V_A}{c^2}\left(\frac{V_o^2}{V_A}\right)^2 \frac{L}{\Omega_{ci}^2 D^3} \tag{30}$$

(assuming $\frac{1}{\bar{D}^2} \gg \frac{V_A^2}{c^2}$).

But this expression is unlike any previous results and differs from that of Drell, et al. by a factor $\frac{1}{3}\left(\frac{V_o^2}{V_A \Omega_{ci} D}\right)^2$.

The shifting back and forth between upper limit estimations and supposedly exact expressions is confusing. Comparing the results of such hybrid calculations to someone else's exact expression seems a dubious procedure.

Our analysis has shown how the excitation of Alfvén waves as the dominant form of radiation depends on the dimension along the line-of-flight of the charge-exchange region between the system and the ionosphere. It was the fact that this dimension is necessarily on the order of meters that enabled us to discard the higher frequency radiation that Barnett and Olbert were then arguing to be important. The vanishingly small dimension used by Dobrowolny and Veltri should lead to the same swamping of the Alfvén band by the lower hybrid band radiation observed by Barnett and Olbert in their orbiting wire calculations, but since Dobrowolny and Veltri are only looking at Alfvén waves they do not deal with this issue.

The Alfvén wave dispersion relation appears naturally in the SAO analysis, and it is clear what the approximations are that it rests on. The SAO analysis is straightforward. Every step of the logic and the calculation is explicitly spelled out. Thus, if there is something wrong with it, it should be possible to specify what it is.

Dobrowolny's colleague Iess did in fact follow through the steps of the SAO analysis. Evidently he found no errors. He has gone through the Dobrowolny/Veltri calculation to his satisfaction as well. The contradictions between the results remain, and they are quite sharp. Since Iess found no mathematical errors in either analysis, Dobrowolny and Iess advanced physical arguments about why the SAO analysis could not be right. They have made their entire case on two points. First, they have checked their calculations and can't find anything wrong with them. This argument could apply to the SAO calculations as well, since they have not pointed out any errors in our analysis. Second, they maintain that their result makes sense physically because there is an explicit quadratic dependence on the velocity of the satellite in their expression for the wave impedance, while the SAO result (they argue) does not go to zero as $v_x$ does.

Note that this is not an argument that the quadratic dependence obtained by them is the correct one, only that a dependence on the velocity that clearly vanishes as the velocity goes to zero is necessary. Should the wave impedance go to zero as the velocity vanishes? Definitely. There is no disagreement on this point. The time variation in the problem is strictly due to the relative motion between the satellite system and the magneto-plasma, so there would be no waves excited by a motionless tethered satellite, should such a system be possible. As we shall demonstrate, the SAO calculation also gives a null result when the velocity is zero. It is also very plausible that the Alfvén wave impedance should <u>not</u> otherwise depend on the velocity, so that the "physical" argument for the Dobrowolny/Veltri result turns out to be weak, to say the least.

We have now arrived at the SAO expression three different ways, but let us examine the two parts of the criticism of the SAO results made by Dobrowolny and his colleagues. First, let us consider their assertion that the

SAO results cannot be correct because they do not go to zero with the system's velocity. This assertion is demonstrably false. Consider our expression for the Fourier transform of $E_y$ (equation (18) in our JGR paper). It contains the factors $\omega\delta(\omega - k_x v_x)$. This is to be integrated over $\omega$. Now the fundamental, defining property of the Dirac delta function is

$$\int f(\omega)\delta(\omega - \omega_o)\,d\omega = f(\omega_o)$$

Thus when we take the inverse Fourier transform to obtain $E_y$, which contains a linear factor of $\omega$, we obtain zero if the satellite velocity is zero. It could not be more evident. All subsequent calculations thus assume that the satellite velocity is not zero, for indeed there are no waves generated in the zero velocity case. Thus for Dobrowolny and his colleagues to take us to task for failing to have an explicit dependence on the satellite velocity in all our subsequent results, which are various limiting cases (all of which assume a non-zero satellite velocity), is unfair.

Iess has maintained that, if one first does the inverse Fourier integration and then takes the limit as $v_x$ goes to zero, the SAO expression for the wave impedance does not go to zero. It is easy to demonstrate that one cannot expect always to get the same result by inverting the order of integration and limit taking. Consider

$$f(\omega, x) = \frac{\omega \ \delta(\omega - \omega_o)}{x^2 + \omega_o{}^2}$$

and take the integral over all $\omega$ and $x$.

$$\int f(\omega, x)\, dx\, d\omega = \int \frac{\omega_o}{x^2 + \omega_o{}^2}\, dx = \pi,$$

which is independent of $\omega_o$. But it has already been assumed that $\omega_o$ is not zero, so that taking the limit as $\omega_o$ goes to zero after the integration is nonsensical. In physical calculations we must always be guided by the physics in deciding "mathematical" questions; as, for example, causality is frequently invoked to determine contour integration paths. Let us apply physical reasoning to the case at hand. What is the source of the $\omega$ factor in our integrand? It corresponds to a time derivative. What is the source of the $\delta(\omega - k_x v_x)$ factor? It comes from the physical condition that the only time variation in the problem is due to the motion of the system with respect to the plasma. Thus taking $v_x$ to be zero before integration is the choice that makes physical as well as mathematical sense.

Now let us turn to the second part of the question. For a given non-zero satellite velocity, what is the proper dependence for the wave impedance on the satellite velocity? We have found that the wave impedance is very similar to that for a bifilar transmission line and have argued that this makes physical sense. If one considers a real bifilar transmission line, the property of the medium between the two wires (which are electrically insulated from each other along their length—as the magnetic field lines effectively are in the ionosphere) which enters into the calculation is the dielectric constant. Now in the low frequency approximation, which is applicable to the case of Alfven waves, the dielectric constant of the plasma (the diagonal, perpendicular components of the dielectric tensor that is) is in fact independent of the frequency. That is, it is independent of the satellite velocity, which is the only source of time variation in the problem under consideration. Thus the Alfvén wave impedance of an electrodynamic tethered satellite system should be roughly independent of the satellite velocity. We can see no argument at all that it should be quadratic in $v_x$, nor has anyone made a physical argument for this particular functional form. We think this quadratic dependence found in the analysis of Dobrowolny and Veltri is just the result of their having made too many dubious, physically inconsistent approximations after a sequence of complicated calculations replete with opportunities for error. We have demonstrated that, using the same starting point as the Italian investigators, we arrive again at our original results following two different routes. Since our analysis is quite transparent and since no one has yet pointed out a flaw in it (within the confines of its numerous assumptions), we see no reason to alter our conclusions.

We suggest that these investigators should focus their attention on the questions of why their results depend on making the Alfvén approximation <u>after</u> complicated calculations and why they differ from the <u>relevant</u> previous calculations, not the Drell results. They should examine the inconsistency that we have pointed out in their analysis.

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## 3.0 LITERATURE SEARCH

The generation and propagation of ionospheric waves from the electrodynamic tethered satellite system is a complex problem. It makes sense to begin by determining what work in related fields might be applicable to the problem or, at least, suggest methods of approach. We have spent considerable time in searching the literature and collecting papers that seemed potentially useful. In this section we will provide a bibliography of publications that are relevant to our problem. This is not an exhaustive list, since many of the listed references also refer to other papers. It is a list of papers that we have either read or scanned, hoping to glean new ideas. Some of them will be be referred to again and again as we progress in our investigation.

The series of papers by C. and P.B. Greifinger have been helpful, even though the special cases they dealt with are not generally applicable to the problem of tether generated waves. Most of the papers are concerned with geomagnetic pulsations and other naturally occurring ionospheric wave phenomena. The problem is to extract useful methods of analysis and apply them to our rather different problem.

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## 4.0 LOCATION OF THE "HOT SPOTS"

Since both Alfvén waves and whistler waves are guided by magnetic field lines, we expect that the field lines will serve to concentrate the electromagnetic energy going into these modes in rather narrow paths through the ionosphere rather than allowing energy to radiate in all directions. This is potentially very beneficial from the standpoint of the strength of the signal arriving at the earth's surface. This focussing of the electromagnetic energy by the geomagnetic field is the basis for the concept of the "hot spot", the area directly under the place where waves from the tethered system arrive at the boundary of the ionosphere and the atmosphere. While much work remains to be done on the physics of wave transmission onto the earth's surface, it is still of some interest to determine the location of the hot spot for various orbital parameters.

SAO has implemented a computer program to trace magnetic field lines; these lines define the trajectories of the Alfvén waves which transport charge. This program is based on the magnetic field model used in *SKYHOOK*. (IGRF adjusted to 1980.0).

The program takes as input: initial latitude, longitude and altitude; and a termination altitude. The field line is traced (in both forward and backward directions) until the termination altitude is reached. This is the altitude at which positive and negative charge flow from opposite ends of a conducting tether would recombine, and is typically taken as 100 km. The latitude, longitude, altitude, cumulative path length, and angle of the magnetic field with the horizontal plane are tabulated along the trajectory; the resulting file may be examined or plotted.

The field line is traced by integrating the ordinary differential equation system:

$$\frac{d\vec{R}(s)}{ds} = \pm \ \hat{e}_B$$

where $s$ is the path length, $\vec{R}(s)$ is the position along the field line, and $\hat{e}_B$ is the unit vector defined by the magnetic field,

$$\hat{e}_B \equiv \frac{\vec{B}}{B}$$

The integration is performed once with the plus sign, and once with the negative sign.

Preliminary results are tabulated and plotted. Note in particular that

- The distance to recombination does depend on both position and altitude.
- In the direction crossing the magnetic equator, the maximum altitude achieved can be quite large.

The first two plots (Figures 4.1 and 4.2) illustrate the paths traced in latitude and longitude by the magnetic field lines going to the ionosphere/atmosphere interface (taken to be at 100 km altitude) starting at the points designated by the solid dots and traveling in both the northward and southward (up into the magnetosphere) directions.

The next four plots show the third dimension — the altitude — of the field line paths plotted versus latitude for the upward (southward) traveling waves generated in the northern hemisphere.

We intend to connect the magnetic field-line tracing program to a satellite tracking program, so that we can map out the path followed by the hot spots in each hemisphere throughout several orbital revolutions for a range of orbital parameters. Coupled with the results of wave propagation calculations, this will give us an estimate of the surface area on the Earth that a given satellite might reach with an electromagnetic signal of a given strength.

From these preliminary results we conclude that, for orbital heights as great as 600 km, the position of the near hot spot, i.e. the one in the same hemisphere as the satellite, is not displaced much in longitude from the position of the satellite itself, while the hot spot's latitude may be shifted by several degrees. The shifts in longitude for the hot spot in the opposite hemisphere from the satellite's are considerably larger, as would be expected because of the greater changes in latitude. To first order these longitude shifts are due to the displacement of the magnetic poles from the Earth's axis of rotation. An interesting point in this preliminary analysis is the 52 degree angle between the magnetic field vector and the horizontal at the atmospheric boundary for a latitude of only 20 degrees at 90 degrees west longitude. This is relevant to the generalization of the Alfven wing results to the case where the magnetic field is inclined with respect to the system's orbital plane.

# NORTH HEMISPHERE TRAJECTORIES

## SKYHOOK MAGNETIC MODEL

Reentry @ 100 km.

| INITIAL | | | NORTH TRAJECTORY | | | | SOUTH TRAJECTORY | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ALT | LAT | LON | LAT | LON | TH* | S* | LAT | LON | TH* | S* | MAX ALT |
| 300 | 20 | 0 | 23.5 | 360 | 30 | 450 | - 4.1 | 3 | 33 | 3030 | 576 |
| | | 90 | 23.3 | 90 | 32 | 430 | - 4.4 | 91 | 32 | 3040 | 583 |
| | | 180 | 22.7 | 181 | 35 | 370 | -14.6 | 174 | 36 | 4600 | 886 |
| | | 270 | 21.4 | 270 | 52 | 260 | -42.4 | 259 | 50 | 10640 | 2655 |
| 300 | 40 | 0 | 41.2 | 360 | 57 | 250 | -27.3 | 13 | 64 | 13180 | 3622 |
| | | 90 | 41.1 | 90 | 60 | 240 | -22.7 | 93 | 60 | 11750 | 3263 |
| | | 180 | 41.3 | 180 | 54 | 250 | -29.2 | 167 | 57 | 13170 | 3554 |
| | | 270 | 40.6 | 270 | 71 | 220 | -62.4 | 246 | 66 | 34150 | 10860 |
| 300 | 60 | 0 | 60.6 | 360 | 73 | 220 | | | | | 17637 |
| | | 90 | 60.4 | 90 | 78 | 220 | | | | | 13323 |
| | | 180 | 60.6 | 180 | 70 | 220 | | | | | 13698 |
| | | 270 | 60.2 | 270 | 84 | 220 | | | | | >36500 |
| 600 | 20 | 0 | 27.3 | 359 | 37 | 1000 | - 8.5 | 4 | 42 | 3820 | 916 |
| | | 90 | 26.9 | 90 | 39 | 960 | - 8.1 | 91 | 39 | 3700 | 896 |
| | | 180 | 26.0 | 181 | 39 | 880 | -17.5 | 173 | 40 | 5220 | 1191 |
| | | 270 | 23.4 | 270 | 54 | 640 | -44.6 | 258 | 52 | 11580 | 3087 |

* TH is the angle of magnetic field with horizontal at reentry.
S is the total path length (km) along trajectory.

Note: Dipole Magnetic North is at Latitude 79 deg, Longitude 70 deg.

Smithsonian Astrophysical Observatory, 11 August 86.

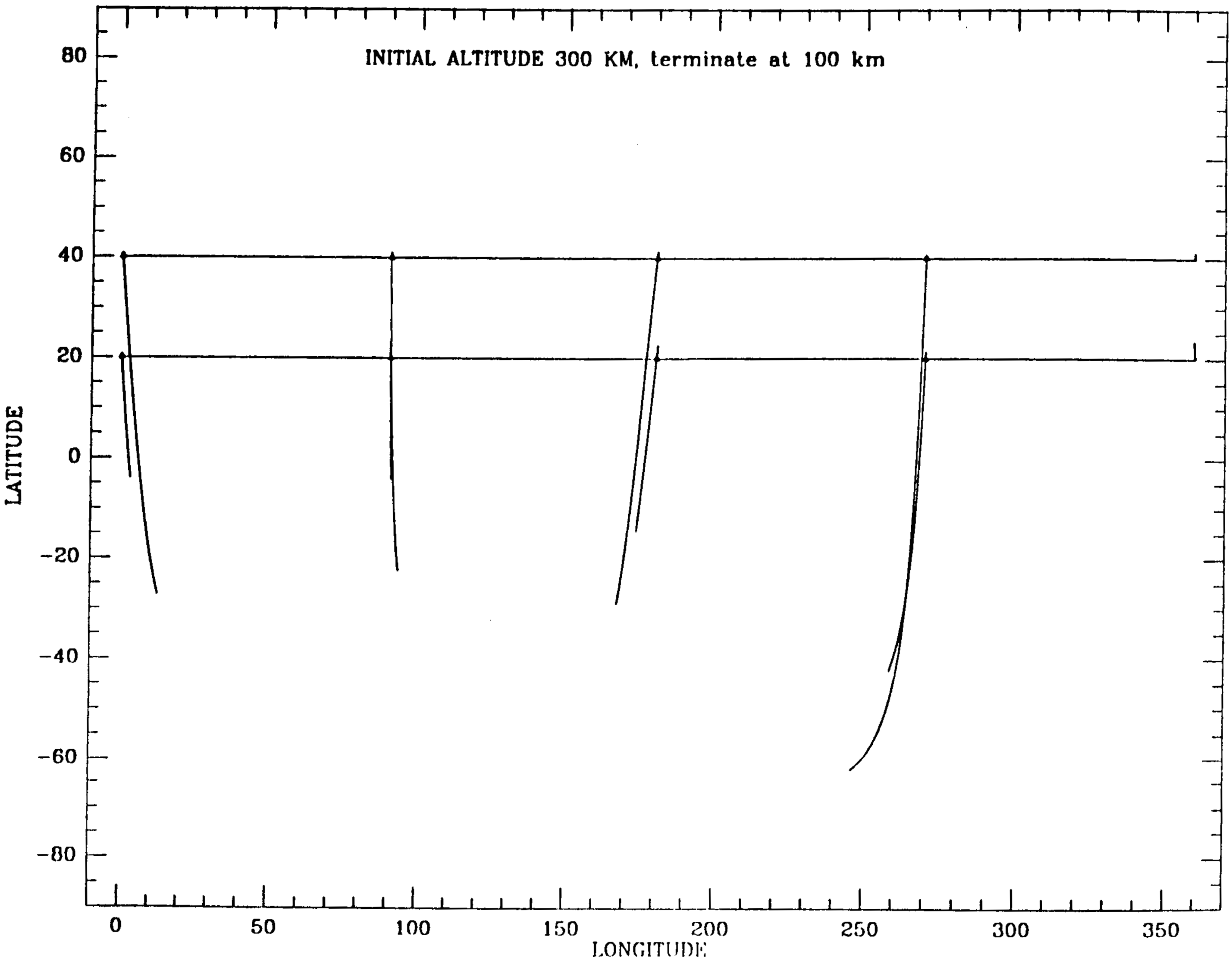



Figure 4.1. Magnetic field line trajectories: latitude vs. longitude.

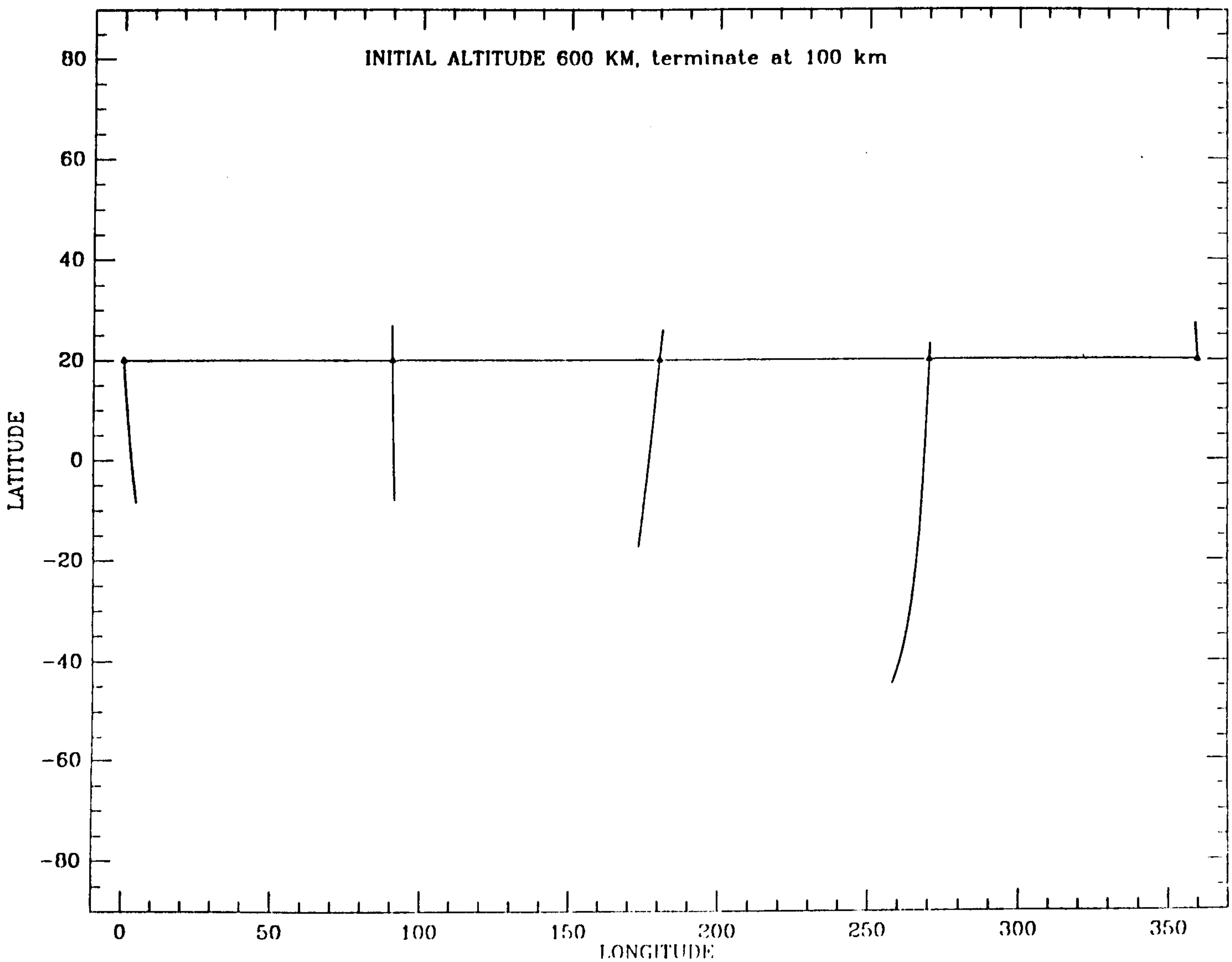



Figure 4.2. Magnetic field line trajectories: latitude vs. longitude.

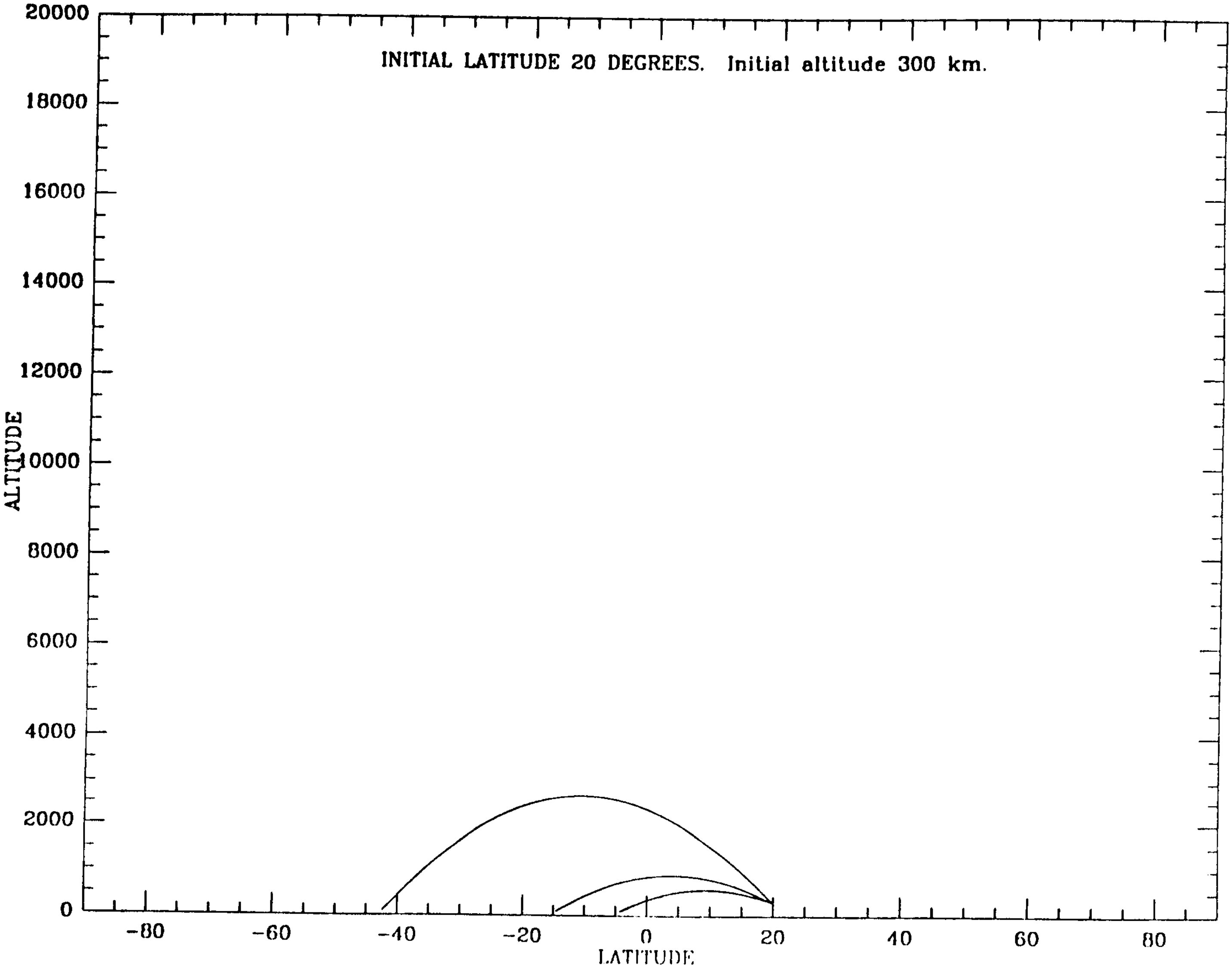



Figure 4.3. Magnetic field line trajectories: altitude vs. latitude.

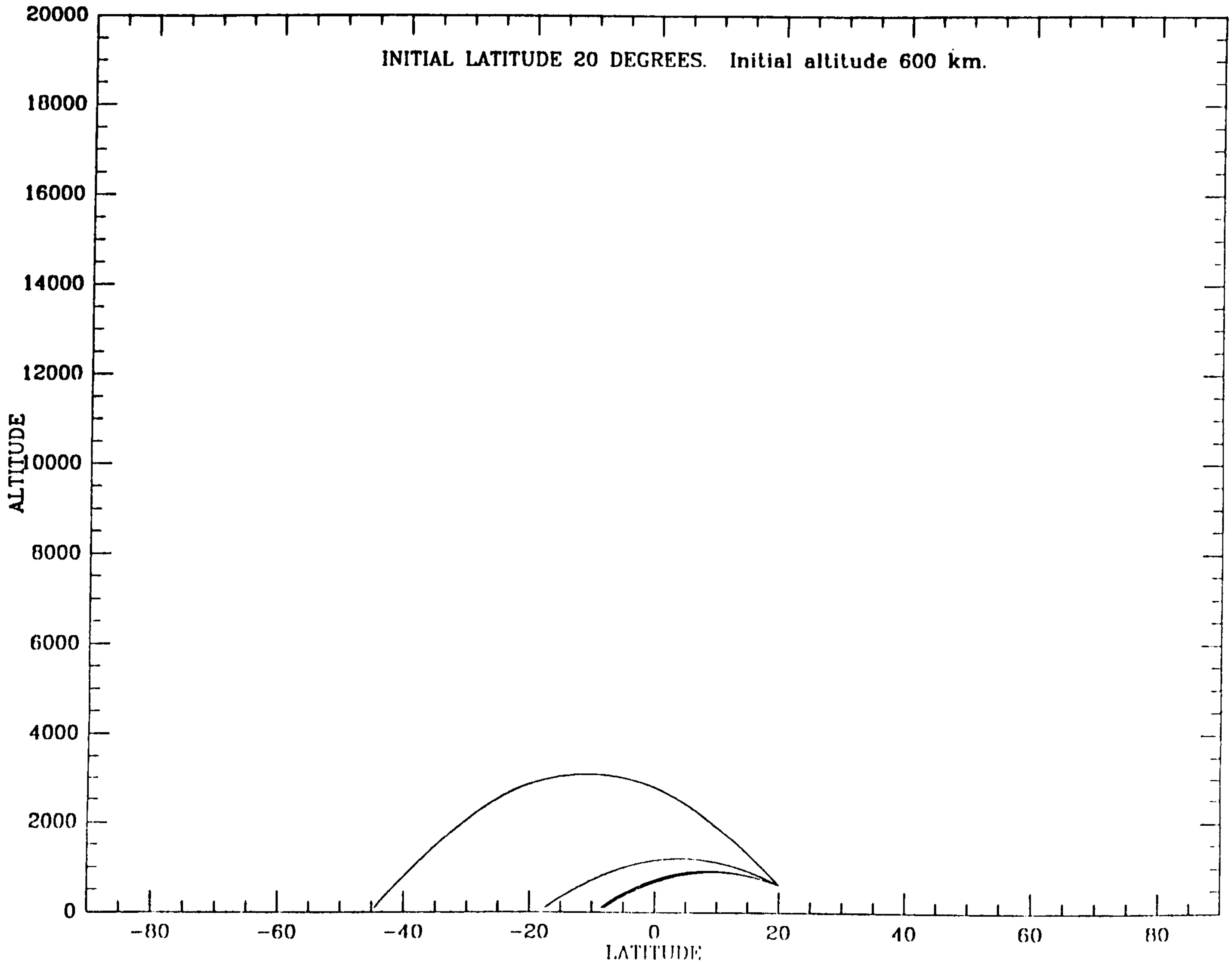



Figure 4.4. Magnetic field line trajectories: altitude vs. latitude.

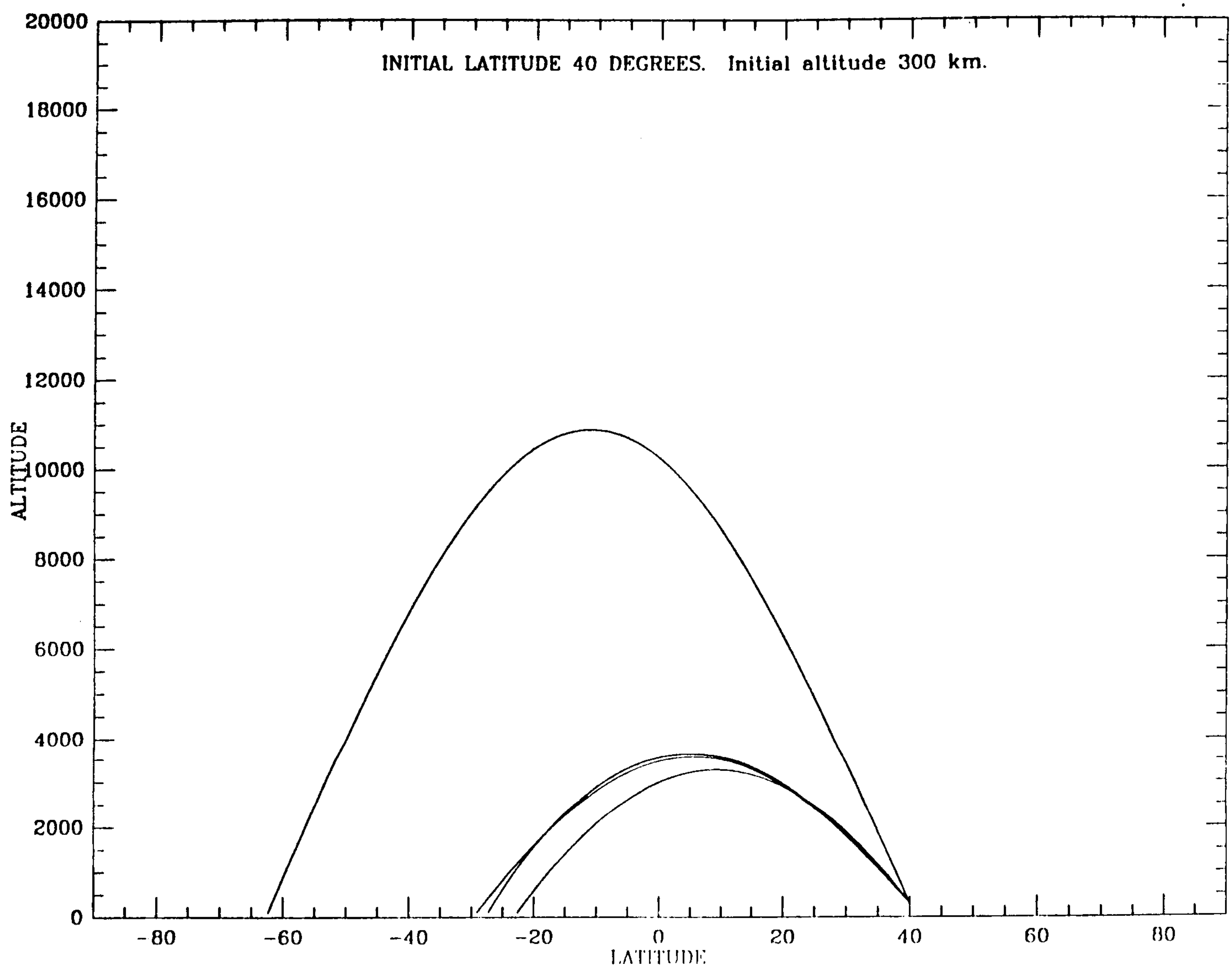



Figure 4.5. Magnetic field line trajectories: altitude vs. latitude.

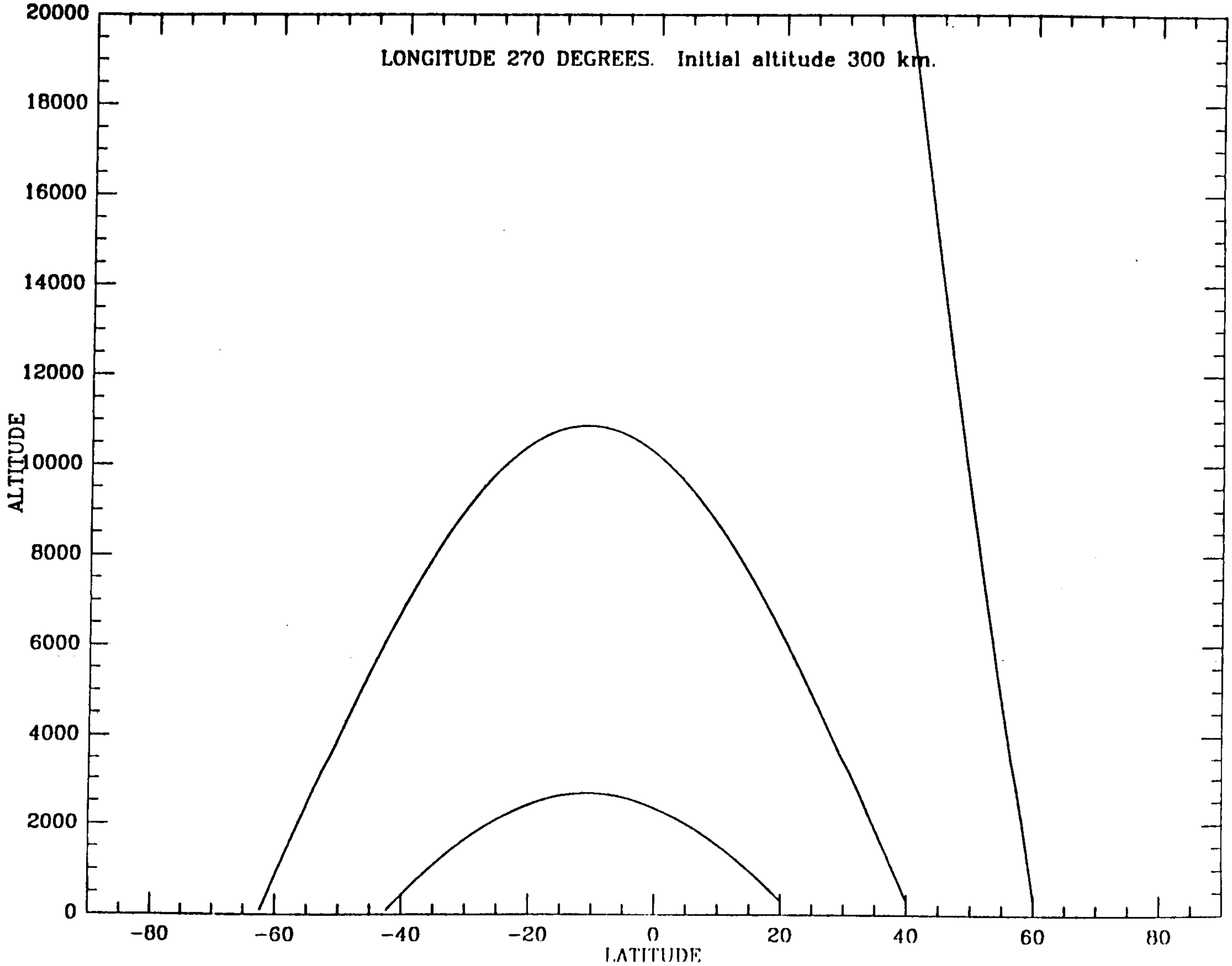



Figure 4.6. Magnetic field line trajectories: altitude vs. latitude.

## 5.0 GENERALIZATION OF THE PREVIOUS RESULTS

Modeling of the electrodynamic tethered satellite system has heretofore been based on some unrealistic assumptions regarding uniformity of the plasma medium and constancy of the tether current. In addition, the simplest relationship has been chosen for the system velocity vector, magnetic field vector, and the vertical. These three directions have typically been taken to define a three-dimensional orthogonal system.

Taking into account the variations in plasma conductivity, density, collision frequencies, etc. in the ionosphere and the existence of the non-conducting atmosphere between the ionosphere and the earth will be the most difficult part of the process of obtaining a more realistic tether model. It will be an iterative project to solve the necessary boundary value problems. We are making progress in this project and expect to have significant results to report by the end of our work.

At this point we have results to report on the generalization of the previous work to the case of arbitrary angles between the vertical, satellite velocity vector, and magnetic field vectors.

First let us consider the case for which the geomagnetic field is not perpendicular to the vertical, but the orbital velocity vector of the system is perpendicular to the plane of the vertical and the magnetic field (see Figure 1). This would correspond to a system at the maximum excursion in latitude for a non-equatorial orbit. Having previously demonstrated the equivalence of an orbiting "ribbon" current distribution and the idealized dumbbell tether current distribution used in the previous analysis (NAG8-551), we can conveniently define the tether current distribution as

$$\vec{j} = \frac{\hat{y}' I}{L_x} \left[ H(x' - L_x/2) - H(x' + L_x/2) \right]$$

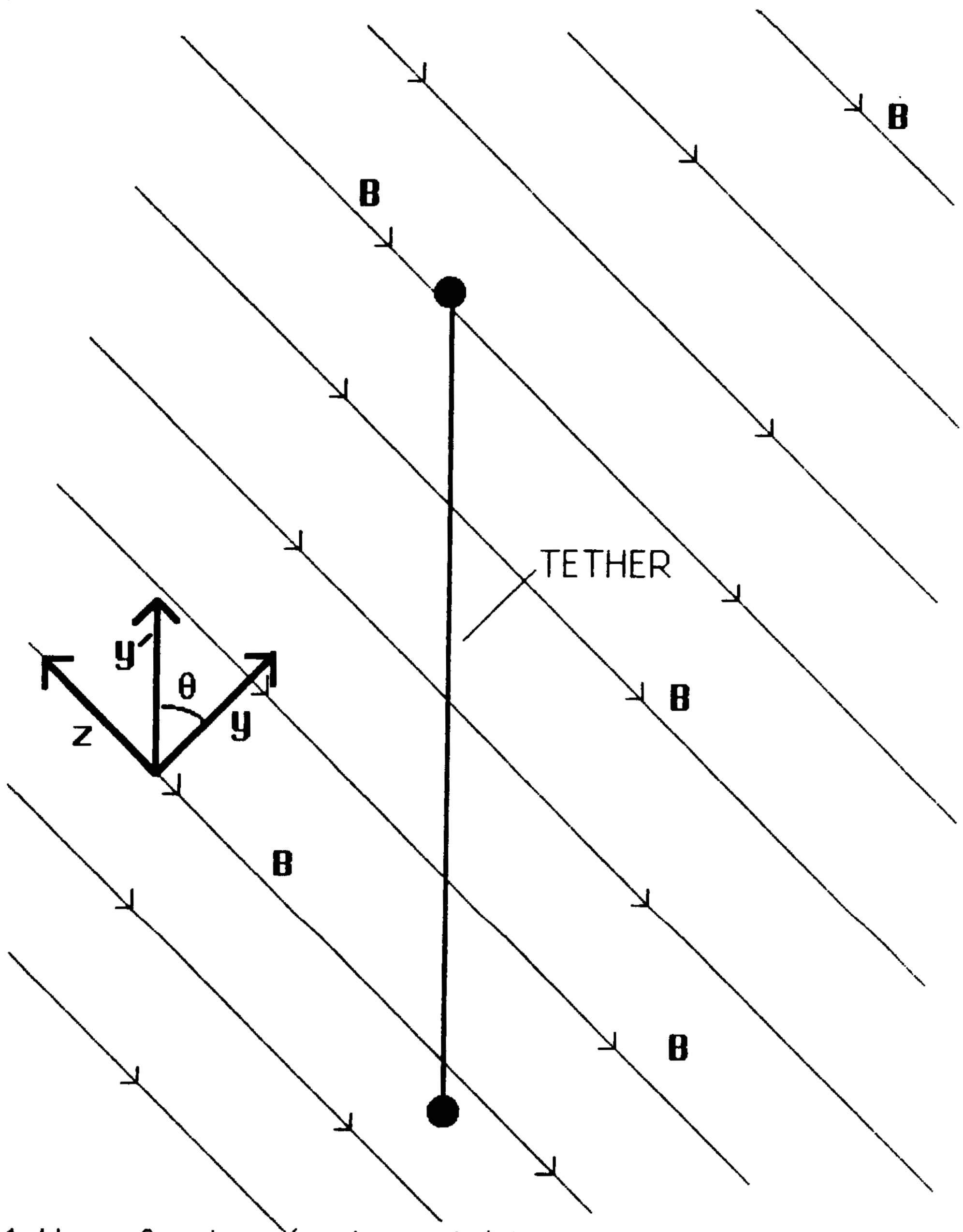
$$\bullet \left[ H(y' - L/2) - H(y' + L/2) \right] \delta(z') \tag{31}$$

where $x' = x - v_x t$ and $\hat{y}'$ lies along the vertical with $y' = 0$ at the middle of the tether, as indicated in Figure 5.1. The $y - axis$, which is orthogonal to $\vec{B}$ (the z axis) and $\vec{v}$ (the x axis) is indicated in Figure 5.1 as well. As in the previous analysis $H(x)$ is the Heaviside function defined by

$$H(x) = 1, \ x \geq 0$$

$$H(x) = 0, \ x < 0$$

We now need $\vec{k} \cdot \vec{j}_k$, where $\vec{j}_k$ is the Fourier transform of the tether current density. This is most conveniently calculated in the $x, y', z'$ co-ordinate system, where




Motion of sytem (and x-axis) in out-of-page direction.
Tether along **y′**-axis.

Figure 5.1. Tethered system with non-horizontal magnetic field.

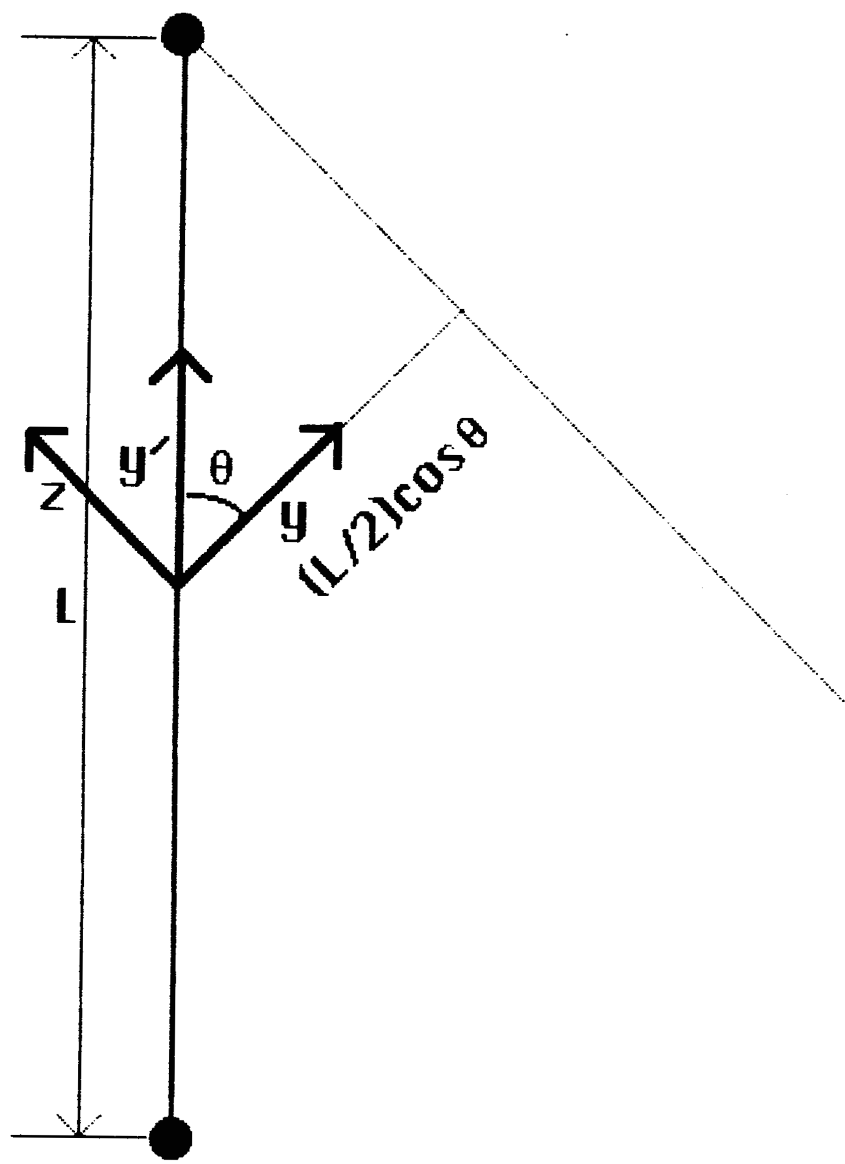



Figure 5.2. Tethered system geometry with non-horizontal magnetic field.

$$\vec{k} \cdot \vec{\jmath}_k = k_{y'} j_{y',k}$$

It is easy to obtain

$$j_{y',k} = \frac{2I}{\pi} \delta\left(\omega - k_x v_x\right) \frac{\sin k_x L_x/2}{k_x L_x} \frac{\sin k_{y'} L/2}{k_{y'}} \tag{32}$$

Thus

$$\vec{k} \cdot \vec{\jmath} = \frac{2I}{\pi} \delta\left(\omega - k_x v_x\right) \frac{\sin(k_x L_x/2)}{k_x L_x} \sin(k_{y'} L/2) \tag{33}$$

Since

$$J_z = \frac{ic^2}{4\pi\omega} k_z \left(\vec{k} \cdot \vec{E}\right) \tag{34}$$

$$\vec{k} \cdot \vec{E} = \frac{4\pi i\omega}{c^2} \frac{\vec{k} \cdot \vec{\jmath}}{(k_z^2 - \omega^2 \epsilon_\perp / c^2)} \tag{35}$$

have been obtained in the previous analysis, we can write the following expression for the Fourier transform of the Alfvén wing or field-line sheet current:

$$J_z = \frac{-2I}{\pi} \frac{k_z}{\left(k_z^2 - \omega^2\epsilon/c^2\right)} \sin\left(\frac{(k_y \cos\theta + k_z \sin\theta)L}{2}\right) \cdot \frac{\sin(k_x L_x/2)}{k_x L_x} \delta(\omega - k_x v_x) \tag{36}$$

which differs from the earlier results for mutually orthogonal vectors only in the argument of one of the sine factors, which takes into account the dip angle of the magnetic field.

The problem is now to obtain the inverse Fourier transform of (11). The integrals over $\omega$ and $k_y$ proceed very much as in the previous analysis, only with the results showing an obvious change due to the inclination of the magnetic field with respect to the horizontal. That is, the charge-exchange regions at the ends of the system are now located at $y = \pm$ (L/2) $\cos\theta$. The integral over $k_z$ also shows the effect of the magnetic field's inclination. The final integral over $k_x$ takes the form

$$\frac{1}{2\pi L_x}\ \delta\Big(y - L\cos\theta/2\Big)\ \Big[2H\Big(z - L\sin\theta/2\Big) - 1\Big]$$

$$\cdot \int_0^{K_0} \frac{\sin(k_x L_x/2)}{k_x}\ \exp\Big(ik_x\big[\,x' + (z - L\sin\theta/2)\alpha\,\big]\Big)\ dk_x \tag{37}$$

$$-\ \frac{1}{2\pi L_x}\ \delta\Big(y + L\cos\theta/2\Big)\ \big[2H(z + L\sin\theta/2) - 1\big]$$

$$\cdot \int_0^{K_0} \frac{\sin(k_x L_x/2)}{k_x}\ \exp\Big(ik_x\big[\,x' + (z + L\sin\theta/2)\alpha\,\big]\Big)\ dk_x$$

where $\alpha = \left(\frac{v_x}{v_A}\right) \frac{1}{\left[\,1-(k_x v_x/\Omega_{ci})^2\,\right]^{1/2}}$

Although this expression looks more complicated than the corresponding expression from the previous analysis, the basic content is the same. The Alfvén wings are seen to be field-line sheet currents at the ends of the system. The sign change in $J_z$ occurs at the charge-exchange interface as before, but these are now located at $z = \pm(L/2)\sin\theta$. The top and bottom wings are connected by the condition of current continuity but otherwise they appear to be independent phenomena generated by the disturbances at their respective ends of the system. Except for the shift in lines of discontinuity in $J_z$ to coincide with those traced by the charge exchange terminals, the Alfvén wing solutions are the same as before. The new solution clearly reduces to the old one when the angle $\theta$ goes to zero.

Another, even more interesting, generalization of the previous analysis results when we allow for an arbitrary angle between the velocity vector of the system and the magnetic field lines. Let us consider the case when the tether lies along the vertical (y) axis and the magnetic field is in the horizontal plane antiparallel to the z-axis. The velocity vector, which lies in the horizontal plane, is allowed to be at an arbitrary angle $\phi$ with respect to the x-axis ((x,y,z) being a mutually orthogonal set of co-ordinate axes), as shown in Figure 5.3.

As before, the dimension of interest is that of the charge-exchange interface perpendicular to the magnetic field lines between the system and the plasma. For a spherical terminating satellite this would be the same dimension $L_x$ as before. The value of the current in the tether, assuming it to be induced strictly by the system's motion, would be reduced by the factor $\cos\phi$ since the induced voltage depends on the component of the velocity perpendicular to $\vec{B}$. Our analysis is done in terms of the tether current $I$ without calculating the value of $I$ beforehand, but it is good to keep this difference in mind.

Once again we make use of the equivalence between an orbiting ribbon of current and the system under study to write the tether current density as

$$j_y = \frac{I}{L_x}\,\delta\Big(z - (v\sin\phi)t\Big)\left[H(y - \frac{L}{2}) - H(y + \frac{L}{2})\right]$$
$$\cdot\left[H(x' - L_x/2) - H(x' + L_x/2)\right] \qquad (38)$$

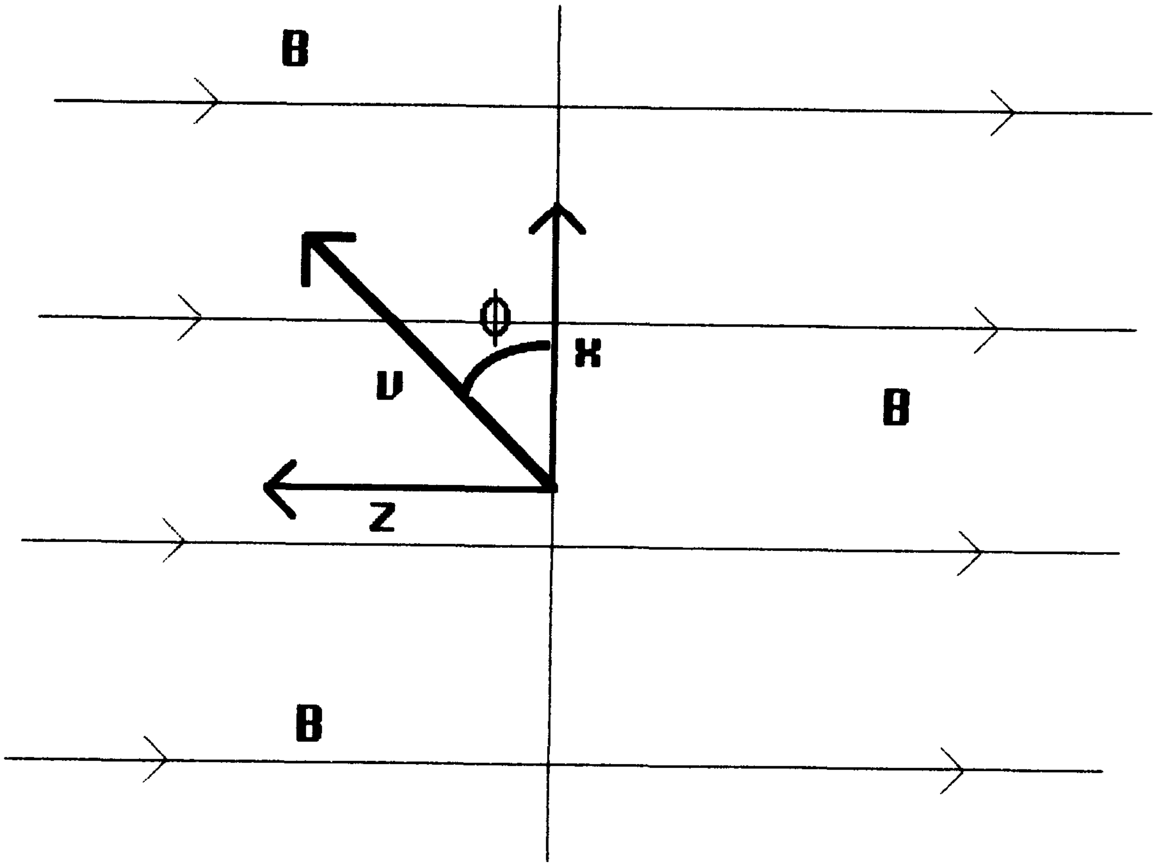



Figure 5.3. Tethered system geometry (viewed from below) for velocity vector not perpendicular to magnetic field.

where $x' = x - (v \cos \phi)t$.

This leads to the Fourier transform result

$$\vec{k} \cdot \vec{j} = \frac{2I}{\pi} \delta(\omega - k_z v \sin \phi - k_x v \cos \phi) \sin\left(\frac{k_y L}{2}\right) \frac{\sin(k_x L_x/2}{k_x L_x} \tag{39}$$

Our expression for the Fourier transform of $J_z$ is identical to the previous one ((14) in our JGR paper) except for the changed argument of the delta function.

The integration over $\omega$ and $k_y$ can proceed as in the JGR article. The singularities on the $k_z$ real axis now occur at solutions of

$$k_z^2 = \left(\frac{V}{V_A}\right)^2 \frac{\left(k_x \cos \phi + k_z \sin \phi\right)^2}{\left(1 - (k_x \cos \phi + k_z \sin \phi)^2 (V/\Omega_{ci}^2)\right)} \tag{40}$$

From our previous results we can infer that, so long as $\phi$ is not too near $\pi/2$, the condition

$$| \, k_x \cos \phi \, | \geq | \, k_z \sin \phi \, | \tag{41}$$

should hold.

Then we obtain

$$k_z = \pm \frac{k_x v}{v_A} \frac{\cos\phi}{\sqrt{1 - \left(k_x v \cos\phi / \Omega_{ci}\right)^2}}$$

for the singularities; i.e., we just replace $V_x$ with $V \cos\phi$ in our previous analysis. The other change occurs in the complex exponentials left in the integrand

$$e^{ik_x(x - v\cos\phi t)} e^{ik_z(z - v\sin\phi t)} \tag{42}$$

After the integration over $k_z$ we obtain

$$J_z(x', y, z') = \frac{1}{2\pi L_x} \Big\{ \left[ \delta(y - L/2) - \delta(y + L/2) \right]$$

$$\cdot \left[ 2H(z') - 1 \right] \cdot \int_0^{K_0} dk_x \, \frac{\sin(k_x L_x/2)}{k_x} \cdot e^{ik_x(x' + \alpha z')} \Big\} \tag{43}$$

where $x' = x - v \cos\phi t$,

$z' = - v \sin\phi t$, and

$$\alpha = \frac{v\cos\phi}{v_A \sqrt{1 - \left(k_x v\cos\phi / \Omega_{ci}\right)^2}}$$

Without getting all the details, we can see that this implies Alfvén wings that look more or less like those shown in Figure 5.4. Seen from above it would appear as a winged structure flying along at an angle rather than "straight ahead."

There are a number of subtleties that would seem to be important, such as the actual shapes of the satellites, however, so that this analysis can only be viewed as a first approximation to an adequate description.

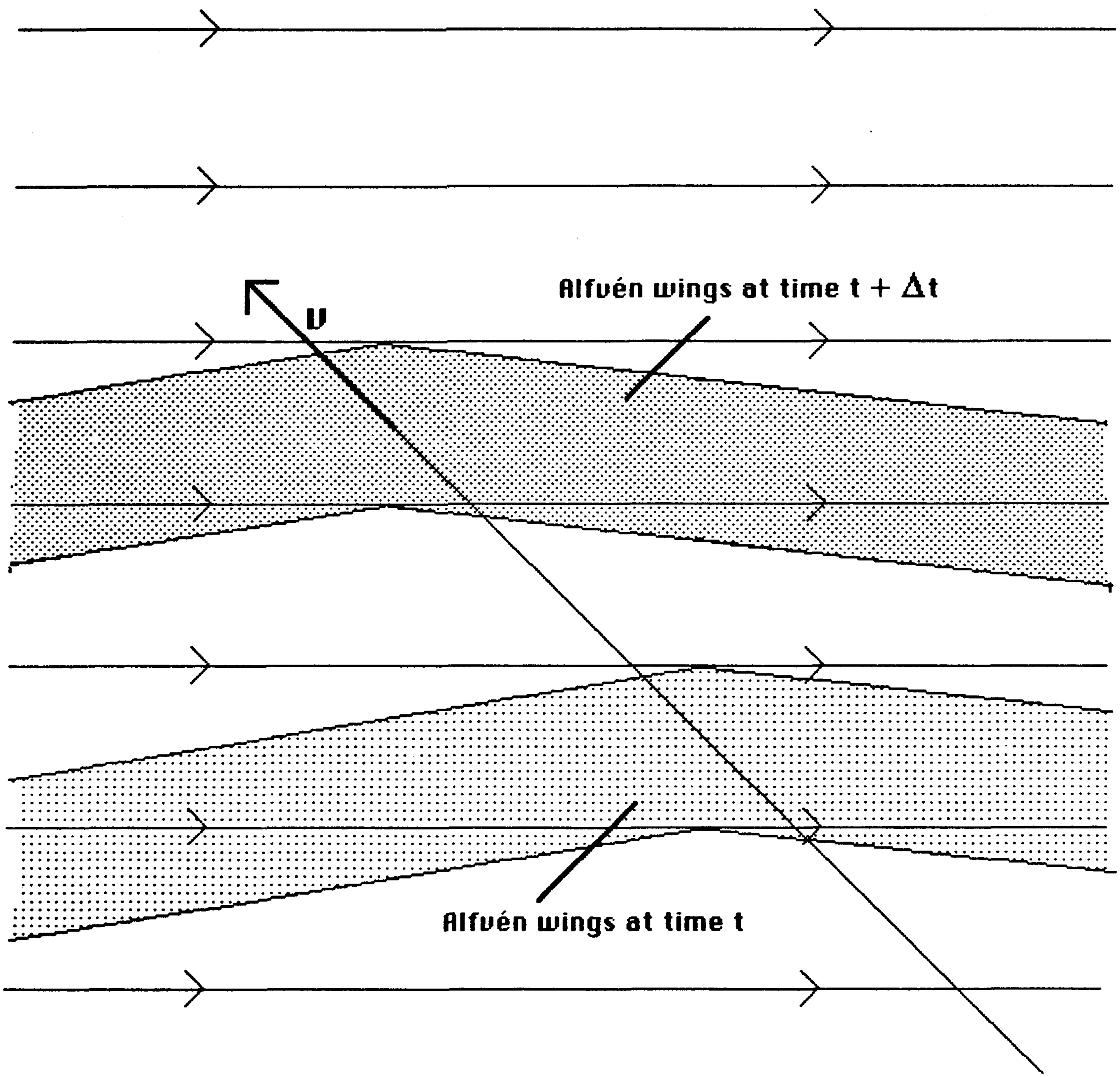



Figure 5.4. Alfvén wing motion (viewed from below) for velocity vector not perpendicular to magnetic field.

## 6.0 ONGOING WORK

An extension of the infinite medium analysis of the ionospheric currents to the case of time-varying tether currents will be carried out in the coming months. For the case of a bounded plasma medium we have begun modeling the tether/ionosphere/atmosphere/ earth system in the following way, which will be made more complex with the addition of more features as we attain success with the simpler model. The ionosphere is being considered as a uniform cold plasma once again. Dissipative effects of collisions are still ignored, clearly not a realistic approximation for the lower levels of the ionosphere. The atmosphere is being considered a vacuum, and the Earth a perfect conductor in the initial model. Figure 6.1 shows this beginning model. The tethered system will first of all be considered as a distant source of Alfven wing wave packets of the type we have described. Both analytical and numerical analyses will be pursued. Preliminary work indicates that the problem is not easy, even in this simplified version. Assuming progress comes fast enough on this part of the analysis, we would like to connect it with the problem of ionospheric currents when the tethered system is not in the far distance, i.e. when it is close enough to the boundary for reflection and the possible creation of ionospheric standing waves to affect the functioning of the tethered system.

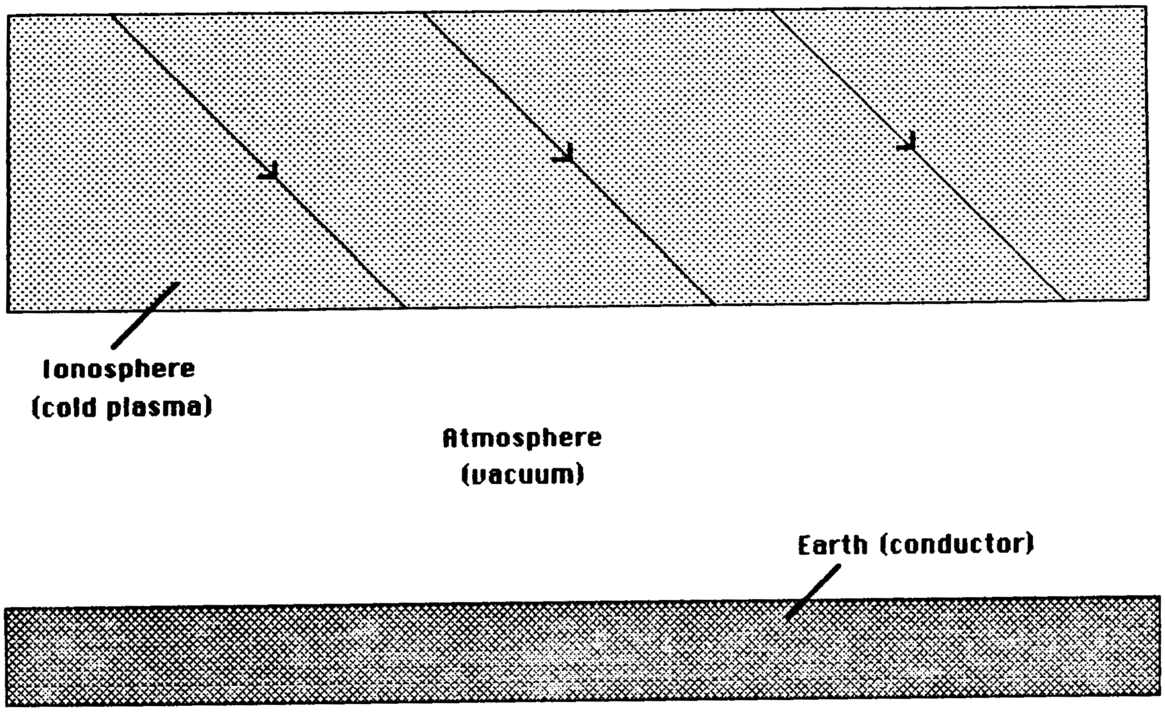



Figure 6.1. Initial model for treating boundary problem of tether waves.